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The Jevons double coincidence condition and local uniqueness of money: An example

Ross M. Starr

Economics Dept. 0508, University of California, San Diego, 9500 Gilman Dr., La Jolla, CA 92093-0508, USA

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ABSTRACT

Jevons's double coincidence of wants condition is derived as the result of household level transaction costs in general equilibrium where N commodities are traded at $(1/2)N(N-1)$ commodity-pairwise trading posts. Each household experiences a set-up cost on entering an additional trading post. Budget constraints are enforced at each trading post separately implying demand for a carrier of value between trading posts, commodity money. General equilibrium consists of prices so that each trading post clears. Existence and local uniqueness of commodity money in equilibrium can follow from the scale economy implied by the household set-up cost.

Andreu Mas-Colell and Esther Silberstein

During the academic year 1978–79, my family and I spent a delightful year living in Berkeley on a Guggenheim Fellowship. Our house on the Northside was a few blocks from Andreu and Esther's and our children were close enough in age to enjoy playing together. There was some discrepancy in our schedules — my workday started around 8:00 AM and ended at 5:00 PM; Andreu's started about noon and ended after we were all asleep. Nevertheless there was ample time for amiable instructive conversations together. It was a particular joy — three decades later — to join in Andreu's birthday party in Barcelona.
In Berkeley in the 1970s, between conversations with Andreu, the focus of my research was to bridge the gap between monetary theory and the Arrow–Debreu general equilibrium model. That's the topic of the article below.

1. Introduction

Jevons (1875) observes:

> "[In monetary] sale and purchase $\cdots$ one of the articles exchanged is intended to be held only for a short time, until it is parted with in a second act of exchange. The object which thus temporarily intervenes in sale and purchase is money. At first sight it might seem that the use of money only doubles the trouble, by making two exchanges necessary where one was sufficient; but a slight analysis of the difficulties inherent in simple barter shows that the balance of trouble lies quite in the opposite direction $\cdots$

E-mail address: rstarr@ucsd.edu.

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> "The first difficulty in barter is to find two persons whose disposable possessions mutually suit each other's wants. There may be many people wanting, and many possessing those things wanted; but to allow an act of barter, there must be a double coincidence, which will rarely happen"

Jevons's statement appears sound, but we should recognize how completely it is at odds with a conventional Arrow–Debreu general equilibrium model, Arrow and Debreu (1954); Debreu (1959). In the Arrow–Debreu model, there are no transaction costs. Each agent has goods he is trying to sell — goods he doesn't want. There is no reason why he should decline — at prevailing prices — to accept one good he doesn't want in exchange for another he doesn't want. In a model with linear transaction costs, he should be willing to make such a trade at a discount reflecting the costs of exchange. And how does money come into the array — a commodity asymmetrically acceptable? If all goods carry zero transaction costs or similar linear transaction costs there is no advantage in accepting money, a commonly traded good (that one doesn't want) in order to retrade it, instead of another that a fellow trader has in excess supply.

This leaves the theorist with the question: what structure of individual incentives or transaction costs results in a general equilibrium pattern of trade that follows the Jevons description? It is well known that the Arrow–Debreu model of Walrasian general equilibrium cannot account for money. Professor Hahn (1982) writes

> "The most serious challenge that the existence of money poses to the theorist is this: the best-developed model of the economy cannot find room for it. The best-developed model is, of course, the Arrow–Debreu version of a Walrasian general equilibrium. A first, and··· difficult··· task is to find an alternative construction without··· sacrificing the clarity and logical coherence ··· of Arrow–Debreu."

This paper pursues a simple example presenting a microeconomic foundation for the *double coincidence of wants* condition on barter trade, with a monetary exception, just as Jevons foresaw. This is a model of monetary trade as the outcome of more elementary cost and utility conditions. The object is to develop a parsimonious example in keeping with the spirit of Walrasian general equilibrium theory that results in commodity money as a medium of exchange; money is to be a result of the model, not an assumption. The double coincidence of wants condition — with monetary trade an exception to the condition — is to be the result of the general equilibrium pricing.

The model posits a pure exchange economy with a trading post structure. For each possible pair of commodities, i and j, there is a separate trading post, denoted $\{i, j\}$ (or equivalently $\{j, i\}$), where the commodities are traded for one another. Transaction costs take a simple non-convex form: each household experiences a utility set-up cost on entering an additional trading post. Thus the household seeks to implement its desired purchases and sales while managing to restrict the number of trading posts where it is active. Transactions are a costly activity (in utility terms, at the household level), and thus they are priced.

The elementary first step is to create a general equilibrium where there is a well-defined demand for a medium of exchange — a carrier of value between transactions. This is arranged by replacing the single budget constraint of the Arrow–Debreu model with the requirement that the typical household or firm pays for its purchases directly at each of many separate transactions at commodity-pairwise trading posts. This represents a formal distinction from the Arrow–Debreu model — there is not merely one budget constraint on an agent but many, one at each commodity-pairwise trading post where he trades.

The household faces a tradeoff. The minimum number of trading posts it can enter is reflected by the number of buying and selling pairs of goods it is interested in; if it has one good for sale and several for purchase, the minimum number of posts it can enter is equal to the number of desired purchases. But those markets may price the household's selling good at a discount. Conversely, if there is a common medium of exchange, commodity money, discounts may be smaller at the monetary trading posts but trading there will add to the number of trading posts the household enters with consequent personal cost.

A well-defined demand for media of exchange arises endogenously as an outcome of the market equilibrium. Money is not a social contrivance or an assumption. It is an outcome of the equilibrium. The price system in equilibrium creates and defines money. The use of commodity money is particularly evident when the structure of demands is characterized by an absence of double coincidence of wants, Jevons (1875). That is, we posit as a starting position endowments and tastes so that no single trading post has complementary demands and supplies at prices implied simply by household preferences and endowments. That the only acceptable trades in general equilibrium — with the exception of monetary trade — are those of complementary demands and supplies is a result of the equilibrium pricing. Media of exchange are characterized as the carrier of value between transactions (not fulfilling final demands themselves).

Formalizing a multiplicity of budget constraints as a basis for modeling monetary structure appears in Hahn (1971, 1973); Starrett (1973); Kiyotaki and Wright (1989); Wallace (1980) and the overlapping generations literature that follows. Kiyotaki and Wright (1989) and the search/random matching model literature that follows emphasizes the pair of trading households as the elementary unit of exchange. The overlapping generations and search/random matching models typically use an infinite horizon and posit fiat money as a bubble, assumed unique and of positive value sustained by expectations of future positive value. Starr (2003a,b, 2008) focuses on the commodity-pairwise trading post structure. In that setting, scale economies in transaction costs generate local uniqueness of commodity money or paper money (the latter valued because of acceptability for tax payments).

2. Commodities and trading posts

In the trading post model, transactions take place at commodity-pairwise trading posts (Shapley and Shubik, 1977; Starr, 2003a,b, 2008; Wallace, 1980) with budget constraints (you pay for what you get in commodity terms) enforced at each post. Prices are quoted as commodity rates of exchange. Market equilibrium occurs when prices at each trading post have adjusted so that all trading posts clear.

A barter equilibrium would consist of an outcome where most goods trade directly for one another, most trading posts becoming active. A monetary equilibrium occurs when most trading posts are inactive: trade concentrates on posts trading each good for the single common medium of exchange.

Let there be N commodities, numbered $1, 2, \cdots N$. They are traded in pairs — good i for good j — at specialized trading posts. The trading post for trade of good i versus good j (and vice versa) is designated $\{i, j\}$; trading post $\{i, j\}$ is the same trading post as $\{j, i\}$.

Thus there are $(1/2)N(N-1)$ possible trading posts.

3. Prices

Goods are traded directly for one another without distinguishing any single good as 'money'. Prices are then quoted as rates of exchange between goods. The present treatment does not distinguish between bid and ask prices. The price of a hamburger might be 5.0 chocolate bars. Note that the price of a chocolate bar then is the inverse of the price of a hamburger. That is, the price of a chocolate bar is 0.2 hamburger.

The price of i at $\{i, j\}$ is $q_i^{\{i,j\}}$. The price of i is in units of j. The price of j is in units of i. The price of j is the inverse of the price of i (and vice versa). That is, $(q_i^{\{i,j\}})^{-1} = q_j^{\{i,j\}}$ is the price of j in units of i. Use of the letter q to denote prices is adopted for consistency with Starr (2003a,b).

4. Budget constraints and trading opportunities

The budget constraint is simply that at each pairwise trading post, at prevailing prices, in each transaction, payment is given for goods received. This condition is embodied in constraint (T.ii) below. The multiplicity of budget constraints is the single most important distinguishing feature of the trading post model. It contrasts with the single lifetime budget constraint of the Arrow–Debreu model. Thus each household trading plan faces $(1/2)N(N-1)$ budget constraints.

5. Households

The distinctive features of the model are (i) transactions exchange pairs of goods, (ii) budget constraints are enforced at each transaction separately, generating a role for a carrier of value between transactions (a medium of exchange) and (iii) transaction costs are incurred at the household level and are assumed to be non-convex (displaying indivisibility). The transaction cost structure is that the household incurs a discrete cost with each trading post it transacts on, a cost that is increasing with the number of trading posts it uses.

For simplicity in the following example, let the population of households be identified with the N commodities. Each household, h will be designated as a type shown by one of $h = 1, 2, \cdots, N$ A household of type h then is endowed with $\mathcal{K}$ units of good h.

There are two special cases:

- A small economy with one household of each type $h = 1, 2, \cdots, N$. In the small economy, there may be non-price rationing of trades in equilibrium. The size of each household's demands causes quantity limits on the trade in individual trading posts and reinforces the implicit scale economy from the set-up character of transaction costs. This leads to local uniqueness of the commodity money in equilibrium.
- A large economy with the same large number of households of each type $h = 1, 2, \cdots, N$. The economy is large enough to overcome the limited size of each household's demands. There is no non-price rationing in equilibrium. Individual households still seek to economize on the number of trading posts they use, but there may be multiple media of exchange in equilibrium. The large numbers assure no non-price rationing in equilibrium at each trading post.

What is a "small" economy? An economy where markets are thin; in this setting it is a model where the scope for arbitrage at prevailing prices is significantly limited by the very small demands prevailing at a single trading post, leading to non-price rationing of sales there. Moving from a theoretical model to interpretation for application, the notion would be that an economy on the scale of North America could probably support several independent currency systems, whereas an economy on the scale of La Jolla California would find that difficult. The ratio of the total volume of trade to the number of distinct commodities appears to be the approximate measure of size.

Households formulate their trading plans deciding how much of each good to trade at each pairwise trading post. This leads to the rather messy notation:

- $b_n^{h,\{i,j\}}$ = planned purchase of good n by household h at trading post $\{i, j\}$.
- $s_n^{h,\{i,j\}}$ = planned sale of good n by household h at trading post $\{i, j\}$.

There is some excess generality in this notation, since the only goods n actually transacted at $\{i, j\}$ are i and j. This point is formalized in condition (T.i) below.

The notation $\oplus$ is defined in the following way. For any $n, j = 1, 2, \cdots, N$, $n \oplus j \equiv n + j$ if $n + j \le N$,or $\equiv n + j - N$ if $n + j > N$. That is, $n \oplus j$ is $n + j$ mod N.

In order to emphasize the absence of double coincidence of wants in the array of original endowments and tastes, we'll assume household h° is endowed with good $h^\circ = 1, 2, \cdots, N$ and prefers $h^\circ \oplus 1$, $h^\circ \oplus 2$, $h^\circ \oplus 3$. Let $N \ge 10$. We can specify utility functions and market prices so that a market-clearing condition is fulfilled: for each good the amount demanded from the market equals the amount supplied to the market. But for any two households, and on any single trading post, there is no double coincidence of wants. Let $\ell > 0$. Then the typical household's utility function is

$$u^h(x^h) = \min[x^h_{h\oplus 1}, x^h_{h\oplus 2}, x^h_{h\oplus 3}] - \ell[\#\{b_i^{h\{i,j\}} \ne 0 | 1 \le i, j \le N\}]^\alpha \quad \alpha > 1, \ell > 0$$

where for household h of type n, for all goods $m \ne n$, $x^h_m \equiv \sum_{\{i,j\}}(b_m^{h\{i,j\}} - s_m^{h\{i,j\}})$.

The concluding term in the expression for u^h reflects the notion that the household incurs a personal transaction cost for each trading post where the household conducts active trade. The cost starts at ℓ for the first post used, and increases per unit with each additional trading post used. Thus the household seeks to manage its trades to satisfy trading needs, represented as the first term on the RHS, while restricting the number of trading posts where it maintains activity. Under the budget balance constraint (T.ii) below, any trading post where h has a buying transaction is also a post where it has a selling transaction. Thus the transaction cost notation in u^h above, counting only trading posts where h buys, includes equivalently an implicit count for posts where h sells.

Given $q_i^{\{i,j\}}, q_j^{\{i,j\}}$, for all $\{i, j\}$, household h then forms its buying and selling plans, in particular deciding which trading posts to use to execute its desired trades. Household h faces the following constraints on its transaction plans:

- (T.i) $b_m^{h\{i,j\}} > 0$ only if $m = i, j$; $s_m^{h\{i,j\}} > 0$ only if $m = i, j$.
- (T.ii) $b_i^{h(i,j\}} \le q_j^{\{i,j\}} \cdot s_j^{h\{i,j\}}$, $b_j^{h\{i,j\}} \le q_i^{\{i,j\}} \cdot s_i^{h\{i,j\}}$ for each $\{i, j\}$.
- (T.iii) Non-negativity of all goods holdings at the conclusion of trade. Thus for each household h of type n, for all goods $m \ne n$, $\sum_{\{i,j\}}(b_m^{h\{i,j\}} - s_m^{h\{i,j\}}) \ge 0$, and for good n, $\sum_{\{i,j\}}(b_n^{h\{i,j\}} - s_n^{h\{i,j\}}) + \mathcal{K} \ge 0$.
- (T.iv) For each household h, $s_i^{h\{i,j\}} \le \mathcal{K}$ for all i, j.

Note that condition (T.ii) defines a budget balance requirement at the transaction level, implying the decentralized character of trade. Since the budget constraint applies to each pairwise transaction separately, there may be a demand for a carrier of value to move purchasing power between distinct transactions. h faces the array of prices $q_i^{\{i,j\}}, q_j^{\{i,j\}}$ and chooses $s_m^{h\{i,j\}}$ and $b_m^{h\{i,j\}}$, $m = i, j$, to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii) and (T.iv). That is, h chooses which pairwise markets to transact in and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints. (T.iv) limits the size of household sales to the scale of endowment. This represents a limit on the scope of arbitrage in a large economy. In the absence of (T.iv) in a large economy, an arbitrageur could undertake arbitrarily large transactions, effectively overcoming the scale economy implicit in the transaction cost structure. The economic rationale for (T.iv) is credit rationing; an excessively large transaction (in this single period economy) necessarily implies an extension of credit during the transaction process to the large transactor.

A *competitive equilibrium* consists of $q_i^{o\{i,j\}}, q_j^{o\{i,j\}}$, $1 \le i, j \le N$, so that:

- For each household h, there is a utility optimizing plan $b_n^{oh\{i,j\}}, s_n^{oh\{i,j\}}$ (subject to T.i, T.ii , T.iii, T.iv), so that for each $\{i, j\}$, $\sum_h b_i^{oh\{i,j\}} = \sum_h s_i^{oh\{i,j\}}$, all $i \ne j$. This concept is subject to non-price rationing in a small economy but not in a large economy.
- An equilibrium is said to be *monetary* with a commodity money, μ, if all transactions are at trading posts including μ and μ is the only good that a household will both buy and sell.

6. A monetary equilibrium

We seek sufficient conditions in an example so that there is an equilibrium pattern of trade where all trade goes through a common medium of exchange, a commodity money. The strategy is to consider three possible patterns of trade by households: barter, monetary trade, arbitrage on the barter markets. We then find trading post break-even prices so that barter and monetary trade are equally successful and so that monetary and arbitrage trade are equally successful. Based on the break-even prices, we can find pricing so that monetary trade is the most rewarding strategy. Barter trade by the household consists of trading on three trading posts: endowed good versus the three desired goods. Monetary trade consists of trading on four trading posts: endowed good versus commodity money, commodity money versus three desired goods. Arbitrage may consist of trade on five trading posts: endowed good versus a second good, second good versus commodity money, commodity money versus three desired goods. In a small economy, quantity constraints may mean that arbitrage including the endowed good requires trade at six trading posts. Arbitrage may alternatively include a non-endowed good: sale of an overpriced good for an underpriced good, sale of the underpriced good for money, repurchase of the overpriced good for money (to deliver the initial sale), purchase of desired goods for money.

In the case where there is a common medium of exchange, commodity money, start by assuming that it trades one-for-one with each of the other goods, fixing money's price at unity. Since all goods enter symmetrically in agents' utility functions, the example below will assume symmetric pricing. Goods 2, 3, 4 are desired by type 1 households: 3, 4, and 5 are desired by type 2 households. $\cdots$ Goods 1, 2, 3 are desired by type N households. Hence we assume $q_i^{\{i,i\oplus 1\}} = q_i^{\{i,i\oplus 2\}} = q_i^{\{i,i\oplus 3\}}$ for $1 \le i \le N$ (with the exception of the case where $i, i\oplus 1, i\oplus 2$, or $i\oplus 3$ is the commodity money and hence the price is unity).

At trading posts $\{i, i\oplus 1\}$, $\{i, i\oplus\}$, $\{i, i\oplus 3\}$, good i is in supply and $i\oplus 1, i\oplus 2, i\oplus 3,$ are in demand. Hence i trades at a discount; $i\oplus 1, i\oplus 2,$ and $i\oplus 3$ trade at a premium.

6.1. *Better than break-even prices*

6.1.1. *Pricing where monetary trade is superior to barter*

Denote the price of the typical non-monetary good owned by household of type i in exchange for a desired good under barter, $q_i^{\{i,i\oplus 1\}}$, by q. By symmetry we take this value to be the same across all non-monetary goods. We'd like to figure out the value of q so that a type i household is better satisfied trading in monetary fashion. In the following expression, the left hand side is the utility of the typical household under barter trade; the right-hand side is the utility of a similar household under monetary trade. The inequality is intended to characterize q° so that monetary trade is preferable to barter:

$$q\frac{\mathcal{K}}{3} - 3^\alpha \ell < \frac{\mathcal{K}}{3} - 4^\alpha \ell$$

Solving for the barter price q° where monetary trade is superior we get that

$$q^\circ < 1 - \frac{3}{\mathcal{K}}(4^\alpha - 3^\alpha)\ell$$

Thus, when barter is sufficiently costly, monetary trade will be superior. Low values of $q_i^{\{i,i\oplus 1\}}$, the barter value of good i, drive trade to using money. How low can q go? If q gets too low, the price will induce arbitrage buying. That limit is investigated next.

6.1.2. *Pricing where monetary trade is superior to arbitrage in a large economy*

In the following expression, the left hand side is the utility of the typical household under monetary trade; the right-hand side is the utility of a similar household performing the following arbitrage: good i for j, j for money, money for $i\oplus 1, i\oplus 2, i\oplus 3$. The size of the arbitrageur's transaction is limited by (T.iv). The inequality is intended to characterize $q^\dagger$, the floor on prices set by the possibility of arbitrage:

$$\frac{\mathcal{K}}{3} - 4^\alpha \ell > \frac{\mathcal{K}}{3q} - 5^\alpha \ell$$

Solving for the lower bound on the barter price q, $q^\dagger$, where monetary trade is superior, we find

$$q > q^\dagger = \left[1 + \left(\frac{3}{\mathcal{K}}(5^\alpha - 4^\alpha)\ell\right)\right]^{-1}$$

Note that this treatment of arbitrage, starting with the endowed good and hence adding only one additional trading post's cost, dominates the alternative of arbitrage with a non-endowed good (adding three trading posts' costs). Hence the value of $q^\dagger$ above is sufficient characterization of arbitrage-free pricing.

6.1.3. *Arbitrage-free pricing in a small economy*

In a small economy matching buyers and sellers is particularly difficult because household quantity limits on the size of desired trades at any single trading post are a binding constraint, leading to non-price quantity rationing.

The small economy has a unique household h of type n. Then when h considers barter trade at trading post $\{n, n\oplus 1\}$, h will supply only $(\mathcal{K}/3)$ of n. An arbitrageur endowed with $n\oplus 1$ considering buying n on $\{n, n\oplus 1\}$ recognizes that he must trade on six different trading posts: $\{n, n\oplus 1\}$ for his arbitrage, $\{money, n\oplus 1\}$ to sell the remainder of his endowment, $\{money, n\}$ to resell the arbitrage purchase, $\{money, n\oplus 2\}$, $\{money, n\oplus 3\}$, $\{money, n\oplus 4\}$ to acquire his desired consumptions. Thus the discount on n at $\{n, n\oplus 1\}$ needs to be sufficiently large to compensate for the additional transaction costs. The quantity x of the abritrageur's endowment of $\mathcal{K}$ that will be utility maximizing for him to expend at price q is characterized by $(1/q)x = (1/2)(\mathcal{K} - x)$. This solves out as $x = (q\mathcal{K}/2 + q)$.

Table 1
Monetary equilibrium trading post prices.

Selling		**1**	**2**	**3**	$\cdots$	n^*	$\cdots$	**N-1**	**N**
Buying	**1**	X	$q_2^{\{1,2\}} = [q^*]^{-1}$	$q_3^{\{1,3\}} = [q^*]^{-1}$	$\cdots$	1	$\cdots$	$q_{N-1}^{\{1,N-1\}} = q^*$	$q_N^{\{1,N\}} = q^*$
	2	$q_1^{\{2,1\}} = q^*$	X	$q_3^{\{2,3\}} = [q^*]^{-1}$	$\cdots$	1	$\cdots$	$q_{N-1}^{\{2,N-1\}} = q^*$	$q_N^{\{2,N\}} = q^*$
	3	$q_1^{\{3,1\}} = q^*$	$q_2^{\{3,2\}} = q^*$	X	$\cdots$	1	$\cdots$	$q_{N-1}^{\{3,N-1\}}$	$q_N^{\{3,N\}} = q^*$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	1	$\vdots$	$\vdots$	$\vdots$
	n^*	1	1	1	1	X	1	1	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	1	$\ddots$	$\vdots$	$\vdots$
	N-1	$q_1^{\{N-1,1\}} = [q^*]^{-1}$	$q_2^{\{N-1,2\}} = [q^*]^{-1}$	$q_3^{\{N-1,3\}}$	.	1	.	X	$q_N^{\{N-1,N\}} = [q^*]^{-1}$
	N	$q_1^{\{N,1\}} = [q^*]^{-1}$	$q_2^{\{N,2\}} = [q^*]^{-1}$	$q_3^{\{N,3\}} = [q^*]^{-1}$	$\cdots$	1	$\cdots$	$q_{N-1}^{\{N,N-1\}} = q^*$	X

This leads to characterizing arbitrage as unprofitable when

$$\frac{\mathcal{K}}{3} - 4^{\alpha}\ell > \frac{1}{3}\left[\frac{1}{q}x + (\mathcal{K} - x)\right] - 6^{\alpha}\ell = \frac{1}{3}\left[\frac{1}{q}\frac{q\mathcal{K}}{2+q} + \left(\mathcal{K} - \frac{q\mathcal{K}}{2+q}\right)\right] - 6^{\alpha}\ell$$

Simplifying this expression to characterize sufficient conditions on q so that arbitrage is unprofitable and monetary trade is superior we have

$$\frac{3(6^{\alpha} - 4^{\alpha})\ell}{\mathcal{K}} > \frac{1-q}{2+q}$$

This expression does not result in a precise value of $q^{\dagger}$, a lower bound on q consistent with prevalence of monetary trade. The RHS is bounded above by 1/2 and the LHS is unbounded. Thus for α and ℓ sufficiently large, monetary trade is overwhelmingly superior to arbitrage. The implication is that monetary trade's superiority is particularly intense in a small economy. In addition, note that in the small economy, it is essential that the monetary instrument be unique in order to match the volume of buy and sell orders for each good on the monetary trading posts. In the large economy, on the contrary, many instruments may act as media of exchange. This suggests that Menger (1892)'s description is particularly appropriate in a small economy:

> why$\cdots$ is$\cdots$ economic man $\cdots$ ready to accept a certain kind of commodity, *even if he does not need it*, $\cdots$ in exchange for all the goods he has brought to market[?]
> Men $\cdots$ exchange goods $\cdots$ for other goods $\cdots$ more saleable$\cdots$ [which] become generally acceptable media of exchange [emphasis in original].

6.1.4. Sustainable pricing in a monetary equilibrium

Thus monetary trade will be sustained where $q^{\circ} > q_i^{\{i, i\oplus 1\}} > q^{\dagger}$. This interval is nonempty in a large economy when

$$1 > 1 - \frac{3}{\mathcal{K}}(4^{\alpha} - 3^{\alpha})\ell > 0 \quad (*)$$

and

$$1 - \frac{3}{\mathcal{K}}(4^{\alpha} - 3^{\alpha})\ell > \left[1 + \left(\frac{3}{\mathcal{K}}(5^{\alpha} - 4^{\alpha})\ell\right)\right]^{-1} \text{(to discourage arbitrage)}$$

or equivalently when

$$\left[1 - \frac{3}{\mathcal{K}}(4^{\alpha} - 3^{\alpha})\ell\right]\left[1 + \left(\frac{3}{\mathcal{K}}(5^{\alpha} - 4^{\alpha})\ell\right)\right] > 1 \quad (**)$$

(*) says that the transaction costs of monetization are not in themselves overwhelming. The inequality (**) will generally be true for $\alpha > 1$ and $\ell > 0$, such that (*) is true.

In a small economy, the less precise sufficient condition is (*) and

$$\frac{3(6^{\alpha} - 4^{\alpha})\ell}{\mathcal{K}} > \frac{1}{2} \quad (***)$$

which will be fulfilled for α, ℓ sufficiently large.

6.2. Monetary equilibrium pricing

Let $q^{\circ} > q^* > q^{\dagger}$. Table 1 presents the equilibrium prices for a monetary equilibrium. Good n^* is the commodity money. The choice of good n^* is arbitrary, since all goods in this example are symmetric, but it is treated asymmetrically as the common medium of exchange. $q_i^{\{i,n^*\}} = 1$ and $q_{n^*}^{\{n^*,j\}} = 1$. The typical price, $q_i^{\{i,j\}}$, is the price for good i at trading post {i, j}. For $i, i \oplus 1, i \oplus 2, i \oplus 3 \neq n^*$, we have $q^\dagger < q^* = q_i^{\{i,i\oplus 1\}} < q^\circ$, $q^\dagger < q^* = q_i^{\{i,i\oplus 2\}} < q^\circ$, and $q^\dagger < q^* = q_i^{\{i,i\oplus 3\}} < q^\circ$. Conversely, $q_{i\oplus 1}^{\{i,i\oplus 1\}} = [q_i^{\{i,i\oplus 1\}}]^{-1}$. At these prices all trade proceeds through the trading posts trading n^*.

The monetary equilibrium pricing in Table 1 reflects the *double coincidence of wants* condition on direct trade. Good i is priced at a discount at trading posts $\{i, i\oplus 1\}$, $\{i, i\oplus 2\}$, $\{i, i\oplus 3\}$ and at a premium at posts $\{i, (i-4)\oplus 1\}$, $\{i, (i-4)\oplus 2\}$, $\{i, (i-4)\oplus 3\}$. The assumed pattern of endowment and preferences means that at no trading post will there be mutually satisfactory barter trades. Double coincidence of wants is assumed absent. Then households do not trade their endowments for a good they desire; equilibrium prices and transaction costs make that unattractive. They trade the endowment for a common medium of exchange and then trade that good for desired consumption. The pattern of trade is an outcome, not an assumption, of the example, reflecting the structure of transaction cost.

7. Conclusion

What was Jevons thinking? The pattern of transaction cost posited here (a set-up cost on each additional trading post a household uses) provides a price-theoretic foundation for Jevons's assumption: "to allow an act of barter, there must be a double coincidence."

The trading post equilibrium with the transaction cost structure posited establishes a well-defined demand for media of exchange as an outcome of the market equilibrium. Media of exchange (commodity monies) are characterized as goods flows acting as the carrier of value between transactions (not fulfilling final demands themselves).

The price system is informative not only about scarcity, desirability, and productivity. It also prices liquidity. Non-monetary goods offered in barter trade have deeply discounted prices. n^* is endogenously determined as the most liquid good. The multiplicity of budget constraints creates the demand for liquidity. The trading post model endogenously generates a designation and flow of commodity money.

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